Substituting R from (2) in (1),

$$W = S\{\sin \varphi(1 - \tan^2 \theta) - 2 \cos \varphi \tan \theta\}. \tag{7}$$

(6) $\div$ (7) gives

$$a \sin^2 \varphi = \frac{b/S}{\sin \varphi(1 - \tan^2 \theta) - 2 \cos \varphi \tan \theta}. \tag{8}$$

For the other extreme angle ψ, $\tan \theta$ changes sign, and we have, by analogy,

$$a \sin^2 \psi = \frac{b/S}{\sin \psi(1 - \tan^2 \theta) + 2 \cos \psi \tan \theta}. \tag{9}$$

(8) $\div$ (9) gives

$$\frac{2 \tan \theta}{1 - \tan^2 \theta} = \tan 2\theta = \frac{\sin^3 \varphi - \sin^3 \psi}{\sin^2 \varphi \cos \varphi + \sin^2 \psi \cos \psi}, \tag{10}$$

giving the required angle.

It may be interesting to note that when the upper edge is smooth, the angle of friction is twice as great as that in (10).

Also solved by A. M. HARDING and G. PAASWELL.

353 (Mechanics).　Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

A uniform beam of oak, 10 feet long, 15 inches deep and 10 inches wide, sustains, in addition to its own weight, a load of 5,000 lbs. placed at the center. Find the greatest bending moment and the greatest stress in the fibers. Take the specific gravity of oak as 0.934.

SOLUTION BY W. J. THOME, Detroit, Michigan.

Let l, b, d be the length, breadth, and depth, respectively, of the beam; M the greatest bending moment; p, the greatest stress in the fibers; and S, the section-modulus of the beam's cross section. Taking the inch and the pound as units and the weight of a cubic foot of water $= 62.5$ pounds, we have

$$M = \tfrac{1}{4}(5000)l + \tfrac{1}{8}\left(bdl \times 0.934 \times \frac{62.5}{1728} \right)l$$

$$= \tfrac{1}{4}(5000)10 \times 12 + \tfrac{1}{8}\left[10 \times 15(10 \times 12) \times 0.934 \times \frac{62.5}{1728} \right]10 \times 12$$

$$= 159121 \text{ inch-pounds.}$$

$$p = M/S = M/\tfrac{1}{6}bd^2 = 6M/bd^2 = 6 \times 159121/10 \times 15^2 = 424 \text{ lbs. per sq. in.}$$

Also solved by PAUL CAPRON.

354 (Mechanics).　Proposed by G. PAASWELL, New York City.

The acceleration of an electric train is constant and equal to a ft. per sec. per sec. Its braking or deceleration is variable and equal to the square root of the velocity. If the distance between stations is 5,000 ft., show that the acceleration must cease and braking ensue when the train is about 960 ft. from the stopping point; also that the maximum velocity attained for a minimum time run is 88 m.p.h. and the time of run is 54 seconds.

SOLUTION BY PAUL CAPRON, U. S. Naval Academy.

Under a constant acceleration, a final speed of 88 mi./hr., or about 129 ft./sec., is acquired, in a distance of 4,040 ft., at the average rate of $64\tfrac{1}{2}$ ft./sec., in about 62.6 secs. The corresponding acceleration is 2.06 ft./sec.^{2}. Evidently the numerical values are incorrect. It will appear further that the distance to be run under acceleration, the greatest velocity acquired and the time of run are all dependent on the acceleration, and that the time of run is a minimum (about 39.15 secs.) when the acceleration is infinite, its duration zero, and the greatest speed about $261\tfrac{1}{4}$ mi./hr.

Let the corresponding values of time in seconds, distance in feet, and speed in feet per second, be t, s, v at any point between stations, and let these letters, with appropriate subscripts, represent

the particular values at P_0, the starting point, P_1, the point where acceleration ceases, and P_2, the stopping point. It is assumed that each of the values t_0, s_0, v_0 and v_2 is zero, and that $s_2 = 5,000$; that the acceleration is a constant, a, from P_0 to P_1, and is equal to $- \sqrt{mv}$ from P_1 to P_2, m having the units ft./sec.^{3}.

From P_0 to P_1, $dv/dt = v(dv/ds) = a$, so that, under the initial conditions, [A] $t_1 = v_1/a$. [B] $s_1 = v_1^2/2a$.

From P_1 to P_2, $dv/dt = v(dv/ds) = - \sqrt{mv}$; whence,

$$\int_{P_1}^{P_2} dt = \int_{P_2}^{P_1} \frac{1}{\sqrt{m}} v^{-1/2} dv; \qquad \int_{P_1}^{P_2} ds = \frac{1}{\sqrt{m}} \int_{P_2}^{P_1} v^{1/2} dv,$$

so that, under the final conditions, $t_2 - t_1 = (2/\sqrt{m})v_1^{1/2}$, $s_2 - s_1 = (2/3\sqrt{m})v_1^{3/2}$, or

$$[C] \quad t_2 = \frac{v_1}{a} + \frac{2}{\sqrt{m}} v_1^{1/2}; \qquad [M] \quad s_2 = \frac{v_1^2}{2a} + \frac{2}{3\sqrt{m}} v_1^{3/2}.$$

From [A], [B], [C], [M]:

$$[N] \quad \frac{1}{a} = \frac{2s_2}{v_1^2} - \frac{4}{3\sqrt{mv_1}}, \qquad [\beta] \quad s_1 = s_2 - \frac{2v_1^{3/2}}{3\sqrt{m}}.$$

$$[\alpha] \quad t_1 = \frac{2s_2}{v_1} - \frac{4v_1^{1/2}}{3\sqrt{m}}, \qquad [\gamma] \quad t_2 = \frac{2s_2}{v_1} + \frac{2v_1^{1/2}}{3\sqrt{m}}.$$

From [γ],

$$dt_2/dv_1 = - \frac{2s_2}{v_1^2} + \frac{1}{3\sqrt{mv_1}} = 0$$

when $v_1 = \infty$, which is impossible by [M], since s_2 is finite, and when $v_1 = \bar{v}_1 = (6\sqrt{m}\, s_2)^{2/3}$. When $v_1 = \bar{v}_1$, moreover, $a = - (6s_2m^2)^{1/3}$, $s_1 = - 3s_2$, so that, as a and s_1 must be positive, dt_2/dv_1 vanishes only under impossible conditions.

Differentiating [N], [α], [β], [γ] and using [M] to eliminate s_2, we find

$$[I] \quad da = 2 \left(\frac{a}{v_1} + \frac{a^2}{\sqrt{mv_1^3}} \right) dv_1, \qquad [II] \quad dt_2 = - \left(\frac{1}{a} + \frac{1}{\sqrt{mv_1}} \right) dv_1,$$

$$[III] \quad ds_1 = - \sqrt{\frac{v_1}{m}}\, dv_1, \qquad [IV] \quad dt_1 = - \left(\frac{1}{a} + \frac{2}{\sqrt{mv_1}} \right) dv_1.$$

Since each of the values a, v_1, and $\sqrt{mv_1}$ is positive, it appears from [I–IV] that as a is increased, v_1 increases and t_1, s_1, t_2 decrease.

From [M],

$$v_1^{3/2} = \frac{3\sqrt{m}}{2} \left[s_2 - \frac{v_1^2}{2a} \right],$$

so that, as $a = \infty$ (s_2 being finite),

$$v_1 \doteq \left(\frac{3}{2} s_2 \sqrt{m} \right)^{2/3},$$

and from [C],

$$t_2 \doteq 2 \sqrt{\frac{v_1}{m}} \doteq \left(\frac{12s_2}{m} \right)^{1/3}.$$

From [A] and [B], $t_1 \doteq 0$ and $s_1 \doteq 0$.

Consequently, the greater the acceleration, the shorter will be the time and distance through which it lasts, the greater will the velocity become, and the sooner will the train arrive. For the quickest run, the train should be shot from a gun with a muzzle velocity of about $261\frac{1}{4}$ mi./hr., braking to start immediately (with an initial intensity of about 1,362 lbs. to the (long) ton).

If $\sqrt{v_1} = x$, we have $x^4 + (4a/3)x^3 = 2as_2 = 10,000\, a$, $t_1 = x^2/a$, $t_2 = (x^2/a) + 2x$, $s_1 = (x^4/2a)$, $s_2 - s_1 = \frac{2}{3}x^3$. [$m = 1$ ft./sec.^{3}, $s_2 = 5,000$ ft.]. From these the following values may be computed:

a	x	v_1 (ft./sec.)	u_1 (mi./hr.)	t_1 (secs.)	(t_2-t_1) (secs.)	t_2 (secs.)	s_1 (ft.)	s_2-s_1 (ft.)
1	9.6825	93.75	63.92	93.754	19.365	113.12	4,394.9	605.1
2	11.278	127.2	86.72	63.595	22.555	86.15	4,044.5	955.5
3	12.264	150.4	102.55	50.13	24.53	74.66	3,770.3	1,229.7
100	18.735	351.0	239.32	3.51	37.47	40.98	616.0	4,384.0
∞	19.573	383.15	261.24	0	39.149	39.15	0	5,000

355 (Mechanics).   **Proposed by HORACE OLSON, Chicago, Ill.**

A solid spheroid, axes a, a, b, is placed with its axis of revolution vertical.   From its highest point a particle is projected horizontally with a speed s.   Where will it leave the spheroid, assuming that it slides on the surface without friction?

SOLUTION BY WILLIAM HOOVER, Columbus, Ohio.

The entire motion is in a vertical central section of the spheroid and all of such sections are equal ellipses.

Let $a^2y^2 + b^2x^2 = a^2b^2$ (1) be the equation of any one of them.

Resolving tangentially,

$$v\frac{dv}{ds} = g\frac{dy}{ds}. \tag{2}$$

Multiplying by ds and integrating,

$$v^2 = 2gy + C. \tag{3}$$

When $v = v_0$, $y = b$, and $C = v_0^2 - 2gb$, and (3) becomes

$$v^2 = v_0^2 - 2g(b - y). \tag{4}$$

If ρ = the radius of curvature, we have, at the point where the particle leaves the curve

$$\frac{v^2}{\rho} = -g\frac{dx}{ds}. \tag{5}$$

Now from (1),

$$\rho = \{(a^2 - b^2)y^2 + b^4\}^{3/2} \div ab^4 \tag{6}$$

and

$$\frac{dx}{ds} = -ay \div \{(a^2 - b^2)y^2 + b^4\}^{1/2}. \tag{7}$$

Substituting (6) and (7) in (5) and reducing,

$$(a^2 - b^2)gy^3 + 3b^4gy + b^4(v_0^2 - 2gb) = 0, \tag{8}$$

a cubic for y.   Now put $a = a/2$, $b = b/2$, $v_0 = s$.

In (8), if $b = a$, and $v_0 = 0$, $y = \frac{2}{3}a$, as is well known for a circle of radius a.

Also solved by PAUL CAPRON.

QUESTIONS AND DISCUSSIONS.

SEND ALL COMMUNICATIONS TO U. G. MITCHELL, University of Kansas, Lawrence.

NEW QUESTION.

35. Is the theorem given below new or has it previously been published?

THEOREM.   *If two parallel planes, π and π', cut sections from a cylindrical surface S and two spherical surfaces S_1 and S_2, and if the sum of the sections of S_2 is equal in area to the sum of the sections of S and S_1, then the part of S_2 included*